Synthetic Scientific Report 2012-2016 Grant 0211/2011

On the implementation of the Grant 145/2012 in the period January 2012 - December 2016.

We present first a synthesis of the achievements and of the ideas developed by the members of the team of the Grant 0211/2011 during the period 2012-2016.

Composition of the team

The team started with the composition: prof.dr. Dan Tiba, prof. dr. Andrei Halanay, prof. dr. Viorel Arnautu and Ph. D. student Diana Merlusca. During 2012, prof. dr. Vasile Dragan was added to the composition of the team and, at the beginning of 2013, Ph. D. student Roxana Nicolai also became a member of the team. Unfortunately, at the beginning of 2014 we lost our colleague Viorel Arnautu who died of a sudden illness at the age of 58. Diana Merlusca defended her Ph.D. thesis in the autumn of 2014 and leaved the team of the grant at the beginning of 2015 (when the extension period started). All these changes in the composition of the Grant team were performed with the observation of all the legal provisions valid in this setting. All the members of the Grant team had important contributions to the results.

Achievements and main ideas

Speaking in a very synthetic manner, the team published 38 research papers, among which 17 in prestigious ISI journals. I also mention that these published works involved cooperation with 14 mathematicians from Romania, Finland, France, Germany, Italy, Brazil, Japan, Bulgaria. More cooperation in the organization of international events, or in the form of reciprocal visits , is reported below. A complete list of publications that mention the Grant is included in this report. All the papers are related to the themes of this Grant, involving control theory and shape optimization problems, with ramifications to free boundary problems, stabilization theory (especially of stochastic systems), general optimization procedures, numerical approximation and numerical experiments. The members of the team were invited for scientific research visits (with external support), for periods from one week to one month, in Univ. of Mulhouse (France), BCAM Bilbao (Spain), Weierstrass Institute (Berlin), Univ. of Jyvaskyla (Finland), Univ. of Sofia (Bulgaria) and in important international meetings to give plenary lectures or to present scientific communications. Numerous such communications were presented in international scientific

meetings, organized abroad or inside Romania, with the support of the Grant as well. All these dissemination activities are according to the planning of the Grant and are reported in detail in each year, in the corresponding Scientific Report on the site of the team, www.dantiba.ro. Moreover, we have organized international specialized minisessions in important International meeting in 2013, 2015 and 2016 (in Klagenfurt, Nice and Perpignan respectively) with the cooperation of M. Sofonea and C. Murea (France) and with an important international participation.

Very prestigious are the nomination of Dan Tiba in the Mathematics Panel of the European Research Council (since 2013) and his election in 2016 as Corresponding member of Accademia Peloritana dei Pericolante, Classe di Scienze, Italy. In 2014 at the International Mathematics Congress in Seul, Dan Tiba chaired an invited session. At the national level, we report the election in 2016 of Dan Tiba as general secretary of the Academy of Romanian Scientists, a prestigious scientific position.

The main research themes of the Grant project appear constantly during the years in all the publications. Concerning shape optimization problems (and similarly the free boundary problems) we summarize the concept of fixed domain methods, that uses essentially the implicit representation of the underlying domains where the problems are defined. The unknown geometry of the problem is reformulated in an analytic way (and in some cases enters as a coefficient in the lower order terms of the partial differential state system) - and the obtained optimization problem is in fact an optimal control problem. The general aim is to reduce shape optimization problems to optimal control problems via a clearly defined and uniform methodology, possible to be applied in a variety of cases. This has the general advantage to replace geometric questions by approximating analytic questions which are favorable from the computational point of view. Our approach has a general character and is advantageous both in their theoretical and numerical aspects. Optimal control theory is a very well established research field of mathematics, while optimal design is a relatively young branch that has not yet found its best solutions and methodology. The relationship with free boundary problems is again a rather new one and seems to be very promising. Control theory is a very rich area and includes as well questions of stability or controllability (both in the deterministic and stochastic cases), or optimization questions and, in the setting of this Grant project, a number of papers were published in such directions too. In the investigation of stability a crucial role is played by the solution of Lyapunov type differential equations and in linear quadratic control problems on infinite time horizon, the stabilizing solution of a Riccati differential equation is involved. In the setting of the Grant, the computation of periodic solutions of Lyapunov differential (or difference) equations was considered and efficient methods were proposed. Similarly, numerical methods for the stabilizing solution of Riccati equations with periodic coefficients were derived. Both solutions of Lyapunov and Riccati equations are used to obtain the state space representation of the optimal filter for filtering problems in the stochastic frame-

work. We have also considered for the first time stabilizing solutions of the Riccati equations with indefinite sign of its quadratic part (this has applications in zero sum linear quadratic differential games). Controllability properties or the lack of such properties were analyzed for parabolic type problems with memory.

We notice that all these areas like geometry, differential equations, optimal control, optimization, numerical analysis and experiments are strongly interrelated and playing essential roles in the solving of shape optimization problems. We consider that the work performed in the setting of this Grant has succeeded a big step forward in the direction of finding a complete and new solution for computational questions and efficient numerical methods in geometric optimization problems and other questions in control theory. Detailed information along these lines and on each published paper can be found in the scientific reports given each year and available on the homepage of the Grant www.dantiba.ro .

We give now the complete list of papers published during 2012-2016 in the frame of the Grant.

LIST OF PUBLICATIONS 2012-2016

ISI papers

1) Dan Tiba, Pekka Neittaanmaki, Fixed domain approaches in shape optimization problems, Inverse Problems, vol.28, pp.1-35, (2012) doi:10.1088/ 0266-5611/28/9/093001, influence score 1, 83

2) Andrei Halanay, Luciano Pandolfi, Lack of controllability of the heat equation with memory, Systems and Control Letters, vol.61, no.10, pp. 999-1002 (2012), influence score 2,03

3) Vasile Dragan, Optimal Filtering for Discrete-Time Linear Systems with Multiplicative White Noise Perturbations and Periodic Coefficients IEEE TRANSACTIONS ON AUTOMATIC CONTROL, DOI: 10.1109/TAC.2012.2215534, vol.58, no.4, (2013), pp.1029-1034. influence score 1,71

4) Peter Philip, Dan Tiba, A PENALIZATION AND REGULARIZATION TECHNIQUE IN SHAPE OPTIMIZATION PROBLEMS, SIAM J. Control Optim., vol.51, no.6, p.4295-4317 (2013). DOI: 10.1137/120892131 Influence score 2,53

5) Vasile Dragan, Samir Aberkane and Ivan G. Ivanov, On computing the stabilizing solution of a class of discrete-time periodic Riccati equations, Int. J. Robust Nonlinear Control (2013). DOI: 10.1002/rnc.3131, impact factor 2,652

6) Andrei Halanay, Cornel Murea, Carmen Safta, Numerical Experiment for Stabilization of the Heat Equation by Dirichlet Boundary Control, Numerical Func-

tional Analysis and Optimization, vol.34, no.12 (2013), p.1317-1327, impact factor 0.542, DOI: 10.1080/01630563.2013.808210

7) Vasile Dragan, Samir Aberkane, H2 optimal filtering for continuous-time periodic linear stochastic systems with state-dependent noise, Systems Control Letters 66 (2014) p.3542, influence score 2,03

8)Vasile Dragan, Stabilizing solution of periodic game-theoretic Riccati differential equation of stochastic control, IMA Journal of Mathematical Control and Information (2014) Page 1-27, doi:10.1093/imamci/dnu026, impact factor 0,967.

9) Vasile Dragan, Robust stabilization of discrete-time time-varying linear systems with Markovian switching and nonlinear parametric uncertainties, International Journal of Systems Science. 45:7, (2014) p.1508-1517 DOI: 10.1137/120892131, impact factor 1,579

10)A, Halanay, L. Pandolfi, Approximate controllability and lack of controllability to zero of the heat equation with memory, JMAA 425 (2015), pp. 194-211, impact factor 1,112.

11)Roxana Nicolai, Dan Tiba, IMPLICIT FUNCTIONS AND PARAMETRIZATIONS IN DIMENSION THREE: GENERALIZED SOLUTIONS, DCDS-A, vol.35, no.6, (2015), pp.2701-2710, impact factor 0.972.

12) Dan Tiba, AN EXAMPLE ON SOME CONDITIONS IN THE CALCULUS OF VARIATIONS, U.P.B. Sci. Bull., Series A, Vol. 77, Iss. 2, 2015, pp.3-8, impact factor 0,405.

13) Vasile Dragan, Samir Aberkane, Ioan-Lucian Popa, Optimal H2 Filtering for Periodic Linear Stochastic Systems with Multiplicative White Noise Perturbations and Sampled Measurements, accepted for publication Journal of the Franklin Institute (2015), DOI: 10.1016/j.jfranklin.2015.10.010 , impact factor, 2,395.

14) Vasile Dragan, Hiroaki Mukaidani - Exponential stability in mean square of a singularly perturbed linear stochastic system with state-multiplicative white-noise perturbations and Markovian switching - *IET Control Theory Appl.*, 2016, Vol. 10, (9), pp. 1040 – 1051, (IF= 1,957).

15) A. Halanay, C.M. Murea, D. Tiba, Existence of a steady flow of Stokes fluid past a linear elastic structure using fictitious domain, J. Math. Fluid Mech., vol.18, DOI 10.1007/s00021-015-fl0247-0 (2016), pp. 397-413. (IF 1,119)

16) Vasile Dragan, Eduardo F. Costa - Optimal stationary dynamic output-feedback controllers for discrete-time linear systems with Markovian jumping parameters and additive white noise perturbations- *IEEE Trans. on Automatic Control*, DOI 10.1109/TAC.2016.2529505, (IF= 2,777).

17) Cornel Murea, Dan Tiba, A direct algorithm in some free boundary problems, Journal of Numerical Mathematics; Online First, DOI 10.1515/jnma-2015-0048 (2016). (IF 0.552)

18) Dan Tiba, Juergen Sprekels, Extensions of the control variational method, Control and Cybernetics, vol. 40, no.4 (2011), pp. 1099-1108., (appeared in 2012).

19) Dan Tiba, Optimal Control Approaches for Some Geometric Optimization Problems, Lecture Notes in Engineering and Computer Science: Proceedings of WCECS 2012, S. I. Ao, Craig Douglas, W. S. Grundfest and Jon Burgstone Editors, Newswood Limited, Hong Kong, (2012) pp.1248-1252, ISBN 978-988-19252-4-4

20) Dan Tiba, Finite element discretization in shape optimization problems for the stationary Navier-Stokes equation,in System Modeling and Optimization: 25th IFIP TC 7 Conference, Berlin, Germany, September 12-16, 2011, Revised Selected Papers, D.Hoemberg and F.Troltzsch Eds.,IFIP AICT 391, Springer, Heidelberg, pp. 437-444 (2013).

21) Peter Philip, Dan Tiba, Shape optimization via control of a shape function on a fixed domain: theory and numerical results, inNumerical Methods for Differential Equations, Optimization and Technological Problems, S.Repin, T.Tiihonen, T.Tuovinen Eds., Comp. Meth. In Appl. Sc. 27, Springer Verlag, Dordrecht (2013), pp.305-320.

22) Dan Tiba, Optimal Control Methods and the Variational Approach to Differential Equations, Lecture Notes in Engineering and Computer Science: Proceedings of IMECS 2013, S.I. Ao, O. Castillo, D. Craig, D.D. Feng, J.-A. Lee, Eds., Newswood Limited, Hong Kong, (2013) pp.127-132, ISBN 978-988-19251-8-3.

23)Vasile Dragan, Toader Morozan, Adrian-Mihail Stoica, H2 OPTIMAL FILTERING FOR DISCRETE-TIME LINEAR STOCHASTIC SYSTEMS WITH PERIODIC COEFFICIENTS AND MARKOVIAN JUMPING, Ann. Acad. Rom. Sci. Ser. Math. Appl. Vol. 5, No.1-2, (2013), pp. 46-64.

24) Vasile Dragan, Toader Morozan, Viorica Ungureanu, SOME LYAPUNOV TYPE POSITIVE OPERATORS ON ORDERED BANACH SPACES, Ann Acad. Rom. Sci. Ser. Math. Appl. Vol. 5, No.1-2, (2013), pp. 65- 107.

25) Dan Tiba, THE IMPLICIT FUNCTION THEOREM AND IMPLICIT PARAMETRIZATIONS, Ann. Acad. Rom. Sci. Ser. Math. Appl. Vol. 5, No.1-2, (2013), pp. 193 - 208

26) C.M. Murea, A. Halanay, Embedding domain technique for a fluid-structure interaction problem, Modeling and Optimization, D. Hmberg, F. Trltzsch, (Eds.), Springer, IFIP AICT 391 (2013), pp. 358-367.

27) V. Dragan, S. Aberkane, I.G.Ivanov Solving discrete-time game theoretic periodic Riccati equations: An iterative procedure, Proceedings to the 12-th European Control Conference, Zurich, 17-19 July 2013, IEEE Catalog Number: CFP1390U-USB, Omnipress, (2013) ISBN: 978-3-9524173-4-8.

28) C.Murea, D.Tiba, A Penalization Method for the Elliptic Bilateral Obstacle Problem, in C. Potzsche et al. (Eds.): CSMO 2013, IFIP AICT 443, pp. 110, Springer, (2014). DOI: 10.1007/978-3-662-45504-3 18

29) C.Murea, A. Halanay, Steady Fluid-Structure Interaction Using Fictitious Domain, in C. Potzsche et al. (Eds.): CSMO 2013, IFIP AICT 443, pp. 110, Springer (2014). DOI: 10.1007/978-3-662-45504-3 12

30) A. Halanay, L. Pandolfi, LACK OF CONTROLLABILITY OF THERMAL SYSTEMS WITH MEMORY, EVOLUTION EQUATIONS AND CONTROL THEORY Volume 3, Number 3, September 2014, pp.485-497. doi:10.3934/eect.2014.3.485

31) R.Nicolai, SHAPE OPTIMIZATION AND THE IMPLICIT PARAMETRIZATION METHOD. APPLICATIONS, in On Form and Pattern, C. Vasilescu, M. Flonta, I. Craciun Eds., Editura Academiei, Bucuresti (2015), p. 83 97.

32) Vasile Dragan, Ivan G. Ivanov, SEVERAL ITERATIVE PROCEDURES TO COMPUTE THE STABILIZING SOLUTION OF A DISCRETE-TIME RICCATI EQUATION WITH PERIODIC COEFFICIENTS ARISING IN CONNECTION WITH A STOCHASTIC LINEAR QUADRATIC CONTROL PROBLEM, Ann. Acad. Rom. Sci. Ser. Math. Appl. Vol. 7, No. 1 (2015), pp.98-120.

33) Dan Tiba, UNILATERAL CONDITIONS ON THE BOUNDARY FOR SOME SECOND ORDER DIFFERENTIAL EQUATIONS, Ann. Acad. Rom. Sci. Ser. Math. Appl. Vol. 7, No. 1 (2015), pp.121-136.

34) Roxana Nicolai, An algorithm for the computation of the generalized solution for implicit systems, Ann. Acad. Rom. Sci. Ser. Math. Appl. Vol. 7, No. 2 (2015), pp. 308-320.

35) Vasile Dragan, Samir Aberkane - Computing the stabilizing solution of a class of generalized differential Riccati equations: A deterministic approximation - 22nd International Symposium on Mathematical Theory of Networks and Systems (MTNS), Minneapolis, USA, 12-15 Julie, Retrieved from the University of Minnesota Digital Conservancy, http://hdl.handle.net/11299/181518, (2016), pag.632–637, ISBN: 978-1-5323-1358-5.

36) Dan Tiba, Boundary Observation in Shape Optimization, in New Trends in Differential Equations, Control Theory and Optimization: Proceedings of the 8th Congress of Romanian Mathematicians, V. Barbu, C. Lefter, I. Vrabie Eds., World Scientific (2016), pp. 301 – 315, ISBN:9789813142855

37) Dan Tiba, A DUALITY APPROXIMATION OF SOME NONLINEAR PDE's, Ann. Acad. Rom. Sci. Ser. Math. Appl., vol.8, no.1, 2016, pp. 68 – 77.

38) Mihaela Roxana Nicolai, High Dimensional Applications of Implicit Parametrizations in Nonlinear Programming, Ann. Acad. Rom. Sci. Ser. Math. Appl., vol.8, no.1, 2016, pp. 44 – 55.

We continue with a detailed description of the activities during the year 2016, that have not been reported before.

Main activities

In the setting of the Grant PN-II-ID-PCE-2011-3-0211, director CS 1 Dr. Dan Tiba (Institute of Mathematics, Romanian Academy, Bucharest) all the activities during 2016 followed the plan from the project submitted during the Grants competition in 2011. Roxana Nicolai, who is a very active member of the team and has already four scientific publications, is now preparing the final form of her thesis and will defend it in the first part of 2017.

The activity in the setting of the Grant, has in 2016, as one of the main results, eight papers published or in an advanced publication stage (among which four are ISI, two are BDI and two are conference articles) by the present four members of the Grant team, alone or in collaboration with well known mathematicians like Hiroaki Mukaidini (Japan), Eduardo Costa (Brazil), Samir Aberkane and Cornel Murea (France). All the articles are strictly related to the research themes of the Grant. Other works (for instance on shape optimization problems with Neumann boundary conditions) are in an advanced elaboration level and will be published next year.

As prestigious international activities related to the Grant, we mention that Dan Tiba continued his work in the setting of the European Research Council, Bruxelles, where he is member of the Mathematics Panel for Consolidator Grants . Moreover, Dan Tiba was elected at the beginning of 2016 Corresponding Member (foreign member) of the Accademia Peloritana dei Pericolanti, Classe di Scienze, Italy. Let me also mention the election of Dan Tiba, in May 2016, for a period of four years, as general secretary of the Academy of Romanian Scientists, a prestigious position in the Romanian scientific life.

In cooperation with prof. dr. C. Murea (Univ. Mulhouse, France) a special session on " Optimal Control and Design with Applications" was organized by Dan Tiba in the setting of the ETAMM 2016 meeting, at the Univ. of Perpignan, France, June 2016. It was attended by more than fifteen participants from four countries.

Invited talks were delivered at Cortona, Italy (June 2016), at the Univ. of Messina, Italy (November 2016) and at the Univ. Craiova (September 2016) by Dan Tiba and at Univ. Sofia (May 2016), Univ. Dobrich (September 2016), Bulgaria, by Vasile Dragan. All these events were supported mainly from external funds.

With the support of the Grant (at least partial), communications were presented as well at conferences in Orlando (AIMS, July 2016) and Boston (SIAM, July 2016) (Dan Tiba), Minnesota (MTNS, July 2016) (Vasile Dragan), Perpignan (ETAMM, May 2016), Dan Tiba, Vasile Dragan and Roxana Nicolai. These dissemination activities are strongly related to the objectives of the Grant.

ISI papers

We discuss now the four ISI papers, the first two are completely published in 2016, the other two are just published Online First in 2016 and can be found online using their DOI indicator.

1) Vasile Dragan, Hiroaki Mukaidani - Exponential stability in mean square of a singularly perturbed linear stochastic system with state-multiplicative white-noise perturbations and Markovian switching - *IET Control Theory Appl.*, 2016, Vol. 10, (9), pp. 1040 – 1051, (IF= 1,957).

In many engineering applications the mathematical model of the controlled process may contain some parasitic parameters such as small time constants, resistances, inductances, capacitances, moments of inertia, small masses, etc. Often such parameters occur in the form of some small coefficients of derivatives of a part of the state variables of the controlled system. In order to avoid the ill conditioning of the numerical computations produced by the presence of the small parameters in the mathematical model, usually one applies the singular perturbation technique to obtain either criteria for stability of considered systems or to design a near optimal control law.

In the deterministic case, the singular perturbation technique means the formal neglecting of the small parameters obtaining in this way a system of lower dimension, known as the reduced system.
The approach from the deterministic case cannot be adapted mutatis mutandis to the stochastic case, when the mathematical model of the controlled system is described by a system of singularly perturbed Ito differential equations.

In this paper, I studied the problem of the exponential stability in mean square for a class of singularly perturbed linear stochastic systems with state-multiplicative white-noise and Markovian jumping parameters. Based on the Lyapunov-type operator, the exponential stability in mean square is discussed intensively. First, after decomposing the full-order stochastic Lyapunov differential equations, a reduced-order system is defined and conditions for this reduced order system to be exponentially stable are derived.

Second, it is shown that if the reduced system is exponentially stable then there exist small perturbation parameters that cause the original stochastic systems to be mean square stable. Moreover, it is also shown that the parameter-independent composite stabilizing controller can be obtained by solving a system of linear matrix inequalities.

2) A. Halanay, C.M. Murea, D. Tiba, Existence of a steady flow of Stokes fluid past a linear elastic structure using fictitious domain, J. Math. Fluid Mech., vol.18, DOI 10.1007/s00021-015-fl0247-0 (2016), pp. 397-413. (IF 1,119)

This work is devoted to the difficult problem of the interaction between a solid elastic structure and a fluid in contact with it. We consider here the stationary case with no slip boundary conditions for the fluid, but the method has potential to be extended to other free boundary problems and optimal design problems. The problem we study is a free boundary problem due to the unknown deformation of the structure. This means that the geometry of the problem is one of its unknowns too, similarly to the case of shape optimization problems. The results we obtain include the theoretical study of the involved nonlinear equation and the development of an approximation method, including numerical examples. One important point is that our methodology enters fixed domain approaches and it is close to the methods investigated for optimal design problems. The main idea is to extend the fluid model to the region corresponding to the solid via a penalization/fictitious domain procedure. Approximation, existence and other properties are studied. This methodology has many advantages both from the computational and the theoretical points of view and is one of the main objectives of this Grant.

3) Vasile Dragan, Eduardo F. Costa - Optimal stationary dynamic output-feedback controllers for discrete-time linear systems with Markovian jumping parameters and additive white noise perturbations- *IEEE Trans. on Automatic Control*, DOI 10.1109/TAC.2016.2529505, (IF= 2,777).

The problem of minimization of the effect of some additive white noise perturbations to a discrete-time system subject to some random switchings driven by a finite state Markov chain was studied under additional assumptions regarding the Markov chain as, for example, the ergodicity of this stochastic process. In this paper, we have solved this optimal control problem without any additional assumption regarding the Markov chain. More precisely, this paper addresses stationary dynamic output feedback control of discrete-time Markovian Jumping Linear Systems (MJLS). A rather general setup is adopted, with indirect and noisy output measurements, infinite time horizon, and not necessarily ergodic Markov chains. The class of admissible controllers consists of all stabilizing systems, dynamic or memoryless, with arbitrary dimension. The quality of stabilization achieved by an admissible controller is measured by a performance criterion described by a long run average cost, under some standard conditions. The optimal controller can be computed off-line resting on two sets of Riccati equations and it only requires the storage of $4N$ matrices where N is the cardinality of the Markov state space. We present an example of a quad-rotor system to illustrate the results and compare the performance of three different control schemes with the proposed one, indicating that it is an interesting, simple alternative for controlling MJLS. In the calculus of variations and in its branches optimal control and shape optimization, necessary (and sufficient if possible) optimality conditions play a fundamental role in the characterization of solutions and their properties, in the design and analysis of efficient algorithms. The article discusses some recent conditions of this type via examples

pointing out that they may be not true, at least in certain situations. Clarifications of this type are strictly necessary in the scientific literature. In the nonconvex case, certain consequences may arise and it is important to avoid possible confusions.

4) Cornel Murea, Dan Tiba, A direct algorithm in some free boundary problems, Journal of Numerical Mathematics; Online First, DOI 10.1515/jnma-2015-0048 (2016). (IF 0.552)

In this paper we propose a new algorithm for the well known elliptic obstacle problem and for parabolic variational inequalities like one and two phase Stefan problem and of obstacle type. Our approach enters the category of fixed domain methods and solves just linear elliptic or parabolic equations and their discretization at each iteration. The unknown geometry (the free boundary) is embedded into an analytic formulation using a penalization approach that involves just the zero order terms in the partial differential equation.
We prove stability and convergence properties. The approximating coincidence set is explicitly computed and it converges in the Hausdorff-Pompeiu sense to the searched geometry. In the numerical examples, the algorithm has a very fast convergence and the obtained solutions (including the free boundaries) are accurate. The methodology can be extended to other free boundary problems and to shape optimization problems, in general to problems involving unknown/moving geometries. The subject of this paper enters the themes of the Grant.

BDI Articles

5) Mihaela Roxana Nicolai, High Dimensional Applications of Implicit Parametrizations in Nonlinear Programming, Ann. Acad. Rom. Sci. Ser. Math. Appl., vol.8, no.1, 2016, pp. 44 – 55.

In shape optimization problems, one approach consists in defining the unknown geometry in an implicit way and reducing the optimization over a family of admissible geometries to optimal control problems over a class of admissible controls. This analytic formulation, with the geometry hidden in an implicit way, has clear advantages both from the theoretical and the numerical points of view since it has a purely functional character and enters the category of fixed domain methods.

An essential step is the implicit parametrization method developed in the previous years by the members of the Grant team. This approach has as well a good impact in other optimization problems like the classical nonlinear programming problem, which is a fundamental model in nonconvex optimization.

Several experiments are reported in this paper, related to the implicit parametrization method and its application in optimization, together with comparisons with the existing methodology and numerical methods (for instance MatLab). The aim is to

test the new ideas in a high dimensional setting that occurs frequently in applications.

6) Dan Tiba, A DUALITY APPROXIMATION OF SOME NONLINEAR PDE's, Ann. Acad. Rom. Sci. Ser. Math. Appl., vol.8, no.1, 2016, pp. 68 – 77.

Shape optimization problems are governed by elliptic systems, linear or nonlinear. During their approximation, a difficult technical step is the discretization of the underlying geometry, which is unknown and changes in each iteration of the algorithm. In particular, the used elements may easily degenerate and the obtained numerical results (via the finite element method) may be meaningless. We propose an alternative approach removing such inconsistencies and allowing more flexibility with respect to the geometry of the problem. We discuss the discretization question for the p - Laplacian equation and a variational inequality associated to fourth order elliptic opera- tors, via a meshless approach based on duality theory. The obtained dual problem is a finite dimensional minimization problem and can be solved efficiently by the computers.

Conference articles

7) Vasile Dragan, Samir Aberkane - Computing the stabilizing solution of a class of generalized differential Riccati equations: A deterministic approximation - *22nd International Symposium on Mathematical Theory of Networks and Systems (MTNS)*, Minneapolis, USA, 12-15 Julie, Retrieved from the University of Minnesota Digital Conservancy, http://hdl.handle.net/11299/181518, (2016), pag.632–637, ISBN: 978-1-5323-1358-5.

In this paper, I have considered the problem of numerical computation of the stabilizing solution of a Riccati differential equation with periodic coefficients arising in connection with a stochastic H_∞ control problem. A globally convergent iterative algorithm is proposed for this purpose. The main idea behind the proposed algorithm is to solve at each main iteration a deterministic H_∞-type Riccati equation. One of the advantages of such a deterministic formulation is that one can use direct methods existing in the deterministic framework, for the numerical solution of the considered class of differential Riccati equations. This paper discusses the implicit parametrization method and continues the investigations from the paper 1) above and from previous works of the authors. The applications in shape optimization are explained and new theoretical applications to implicitly defined equations (for instance of Lagrange type) or to variational methods associated to implicit functions, are introduced and analyzed. Numerical examples are also provided. A variational approach is discussed as well.

8) Dan Tiba, Boundary Observation in Shape Optimization, in New Trends in Differential Equations, Control Theory and Optimization: Proceedings of the 8th Congress of Romanian Mathematicians, V. Barbu, C. Lefter, I. Vrabie Eds., World Scientific (2016), pp. 301 – 315, ISBN:9789813142855

We discuss the case of shape optimization problems with boundary cost functional. We investigate theoretical properties and an important tool is the implicit parametrization theorem. It is to be underlined that boundary observation is rarely considered in optimal design problems due to computational difficulties related to the unknown geometries and to the regularity requirements. The methodology that we have developped in the setting of this Grant allows a new approach that removes such unclear points and has a fixed domain character. From the numerical point of view, this is an advantage with respect to other known methods.

An important theoretical result is the derivation of a new formula for directional derivatives in shape optimization problems, in dimension two. It may be applied in gradient methods for shape optimization problems.