Scientific Report

On the implementation of the Grant 145/2012 in the period December 2014 - December 2015

A)Team organization and main activities

In the setting of the Grant PN-II-ID-PCE-2011-3-0211, director CS 1 Dr. Dan Tiba (Institute of Mathematics, Romanian Academy, Bucharest) the activities of the year 2015 followed the plan from the project submitted during the Grants competition in 2011. The members of the team are CS 1 Dr. Dan Tiba (director), Prof. Dr. Andrei Halanay, (Univ. Politehnica, Bucharest). Starting with April 2012, CS 1 Dr. Vasile Dragan from the Institute of Mathematics, Romanian Academy, Bucharest, is as well a very active member of the Grant team. In the autumn of 2012, Roxana Nicolai (who finished her Master in July 2012) started her PhD studies at the Institute of Mathematics, Romanian Academy, Bucharest, supervisor CS 1 Dr. Dan Tiba and became a member of the Grant team from 2013. Unfortunately, we lost our colleague, Prof. V.Arnautu from the Univ. of Iasi, who deceased on January 4-th 2014, after a sudden illness. Dr. Merlusca Diana who defended her Ph.D. Thesis in September 2014 did not extend her participation in the Grant team after the end of 2014.

The activity in the setting of the Grant has in 2015, as one of the main results, nine papers published or in an advanced publication stage (among which five are ISI, three are BDI and one is a conference article) by the members of the Grant team, alone or in collaboration with well known mathematicians as Luciano Pandolfi (Italy), Samir Aberkane and Cornel Murea (France), Ivan Ivanov (Bulgaria). All the articles are strictly related to the research themes of the Grant. Other articles are in an advanced stage of elaboration, and they will be reported in the next step of the Grant.

As prestigious international activities related to the Grant, we mention that Dan Tiba continued his work in the setting of the European Research Council, Bruxelles, where he is member of the Mathematics Panel for Consolidator Grants .

In cooperation with prof. dr. M. Sofonea (Univ. Perpignan, France) a special session on "Modelling and Control in Contact Mechanics" was organized in the setting of the IFIP 2015 meeting, at the Univ. of Nice, France, June 2015. It was attended by more than twenty participants from seven countries.

Invited talks were delivered at the Romanian Academy, Bucharest (March 2015), at the Univ. of Jyvaskyla, Finland (April 2015) and at the Technical Univ. Iasi

(November 2015). All these events were supported mainly from external funds.

With the support of the Grant, the members of our team attended and delivered lectures at the IFIP 2015 meeting in Nice, June 2015 (Dan Tiba, Andrei Halanay and Roxana Nicolai), at the SIAM meeting in Paris, July 2015 (Dan Tiba) and at Congress of Romanian Mathematicians, Iasi, June 2015 (Andrei Halanay and Vasile Dragan). These dissemination activities are strongly related to the objectives of the Grant.

Prof. Dr. J. Sprekels (Univ. Humboldt and Weierstrass Institute, Berlin) was invited for a scientific visit in March 2015 and gave a lecture in the Monthly Seminar of the Institute of Mathematics of the Romanian Academy. We had very useful discussions on our research projects and on extending our cooperation in the future.

B) ISI papers

We discuss now the five ISI papers, the first three are published in 2015, number 4 is accepted and can be found online using its DOI and number 5 has already a report with just one technical observation and will submit soon the revised version to JMFM.

1) **IMPLICIT FUNCTIONS AND PARAMETRIZATIONS IN DIMENSION THREE: GENERALIZED SOLUTIONS**, Dan Tiba, Roxana Nicolai, DISCRETE AND CONTINUOUS DYNAMICAL SYSTEMS Volume 35, Number 6, June 2015, p. 2701 - 2710, doi:10.3934/dcds.2015.35.xx, impact factor 0,972.

Fixed domain methods in shape optimization involve the use of implicit representations of unknown domains. In problems with Dirichlet boundary conditions, there are already techniques to handle such approaches. In the case of Neumann (or other types of boundary conditions) a very detailed knowledge of the unknown boundaries is necessary. This paper discusses a special new approach, based on ordinary differential equations, in order to obtain the desired data. Starting from the implicit equation (in dimension three) an iterated Hamiltonian system is introduced that provides the local solution of the equation in a parametric form. This allows to handle for instance the normal derivatives on the unknown boundaries. The approach can be extended to the critical case, that could not be handled up to now in the mathematical literature. A notion of generalized solution is defined for singular equations. Several numerical examples, both in the classical and in the critical cases, are indicated. The article extends previous work of the authors and will be generalized to general implicit systems in arbitrary dimension, in future papers.

2) **Approximate controllability and lack of controllability to zero of**

the heat equation with memory, Andrei Halanay, Luciano Pandolfi Journal of Mathematical Analysis and Applications (2014), Volume 425, Issue 1, 1 May 2015, Pages 194 - 211, impact factor 1,112.

In this paper we consider the heat equation with memory in a bounded region $\Omega \subset \mathbb{R}^d$, $d \geq 1$, in the case that the propagation speed of the signal is infinite (i.e. the Colemann-Gurtin model). The memory kernel is of class C^1. We examine its controllability properties both under the action of boundary controls or when the controls are distributed in a subregion of Ω. We prove approximate controllability of the system and, in contrast with this, we prove the existence of initial conditions which cannot be steered to hit the target 0 in a certain time T, of course when the memory kernel is not identically zero. In both the cases we derive our results from well known properties of the (memoryless) heat equation.

The results of this paper enter the objectives of the present Grant.

3)D. Tiba, AN EXAMPLE ON SOME CONDITIONS IN THE CALCULUS OF VARIATIONS, U.P.B. Sci. Bull., Series A, Vol. 77, Iss. 2, 2015, p.3 - 8, impact factor 0,405

In the calculus of variations and in its branches optimal control and shape optimization, necessary (and sufficient if possible) optimality conditions play a fundamental role in the characterization of solutions and their properties, in the design and analysis of efficient algorithms. The article discusses some recent conditions of this type via examples pointing out that they may be not true, at least in certain situations. Clarifications of this type are strictly necessary in the scientific literature. In the nonconvex case, certain consequences may arise and it is important to avoid possible confusions.

4) **Optimal H2 Filtering for Periodic Linear Stochastic Systems with Multiplicative White Noise Perturbations and Sampled Measurements**, Vasile Dragan, Samir Aberkane, Ioan-Lucian Popa, Journal of the Franklin Institute, doi: 10.1016/j.jfranklin.2015.10.010, 23p., impact factor 2,395

The main goal in this research paper is to find conditions for the existence and to provide efficient procedures for numerical computation of the unique bounded solution of a class of matrix linear differential equations of Lyapunov type with periodic coefficients and of bounded and stabilizing solution of a Riccati differential equation with periodic coefficients. In 2015, I have studied Lyapunov type differential equations with finite jumps of the form:

$$\begin{aligned}\dot{X} &= A_0(t)X(t) + X(t)A_0^T(t) + A_1(t)X(t)A_1^T(t) + S(t),\\ &\quad kh < t \le (k+1)h\\ X(kh_+) &= \sum_{j=0}^{1} A_{dj}(k)X(kh)A_{dj}^T(k) + S_d(k), \quad k \in \mathbb{Z} \end{aligned} \tag{1}$$

and Riccati type differential equations with finite jumps of the form:

$$\begin{aligned}\dot{Y}(t) &= A_0(t)Y(t) + Y(t)A_0^T(t) + S(t), \quad kh < t \le (k+1)h\\ Y(kh_+) &= A_{d0}(k)Y(kh)A_{d0}^T(k) - A_{d0}(k)Y(kh)C_d^T(k)(R_d(k) +\\ &+ C_d(k)Y(kh)C_d^T(k))^{-1}C_d(k)Y(kh)A_{d0}^T(k) + S_d(k), k \in \mathbb{Z}.\end{aligned} \tag{2}$$

In (1) and (2) $A_j(\cdot), S(\cdot)$ are continuous matrix valued functions which are periodic of period $\theta > 0$ and $\{A_{dj}(k)\}_{k\in\mathbb{Z}}$, $\{C_d(k)\}_{k\in\mathbb{Z}}$, $\{R_d(k)_{k\in\mathbb{Z}}\}$, $\{S_d(k)\}_{k\in\mathbb{Z}}$ are periodic matrix valued sequences of period $\theta_d \ge 1$. One assumes that the periods θ and $\theta_d h$ are rational dependent.

We have proved that if the zero solution of the homogeneous differential equation associated to (1) is exponentially stable, then the affine differential equation (1), has a unique bounded on $\mathbb{R}$ solution and additionally this solution is periodic of period $\tilde{\theta}$, where $\tilde{\theta} = \theta q_d = q\theta_d h$, q, q_d being coprime natural numbers defined by $\frac{\theta}{\theta_d h} = \frac{q}{q_d}$.

Also, we have shown that the Riccati type differential equation with finite jumps (2) has a unique bounded and stabilizing solution which is a periodic function with period $\tilde{\theta}$. For the numerical computation of the stabilizing and periodic solution of (2) I propose a Newton-Kantorovich type iterative method.

The obtained results were included in the paper "Optimal H_2 Filtering for Periodic Linear Stochastic Systems with Multiplicative White Noise Perturbations and Sampled Measurements" accepted for publication to Journal of Franklin Institute (IF = 2,395). In this work we have shown how we can use the unique periodic solution of a differential equation with finite jumps of type (1) and the stabilizing and periodic solution of a Riccati type differential equation with finite jumps of type (2) to derive the state space representation of the optimal filter in a filtering problem with sampled measurements. We refer to the problem of filtering (estimation) of a signal $z(t), t \in [0, \infty)$ generated by a dynamic system modeled by a Ito differential equation with periodic coefficients. Even if the signal which must be estimated is a continuous time one, it is assumed that the measurements involved in the filtering process are available at discrete time instances $t_k = kh, k = 0, 1, \ldots$. Using the periodic solutions of the equations of type (1) and (2), explicit formulae of the coefficients of the optimal filter are provided.

5) **Existence of a steady flow of Stokes fluid past a linear elastic structure using fictitious domain**, Andrei Halanay, Cornel Marius Murea and Dan Tiba, submitted to J. of Mathematical Fluid Mechanics, impact factor 1,119

This work is devoted to the difficult problem of the interaction between a solid elastic structure and a fluid in contact with it. It is a free boundary problem due to the deformation of the structure. This means that the geometry of the problem is one of its unknowns too, similarly to the case of shape optimization problems. The results we obtain include the theoretical study of the involved nonlinear equation and the development of an approximation method, including numerical examples. One important point is that our methodology enters fixed domain approaches and it is close to the methods investigated for optimal design problems. This methodology has many advantages and is one of the main objectives of this Grant.

C) BDI Articles

6) **SEVERAL ITERATIVE PROCEDURES TO COMPUTE THE STABILIZING SOLUTION OF A DISCRETE-TIME RICCATI EQUATION WITH PERIODIC COEFFICIENTS ARISING IN CONNECTION WITH A STOCHASTIC LINEAR QUADRATIC CONTROL PROBLEM**, Vasile Dragan and Ivan G. Ivanov, Ann. Acad. Rom. Sci. Ser. Math. Appl. Vol. 7, No. 1 (2015), p.98 - 120.

Several procedural issues regarding the implementation of some iterative methods for numerical computation of the bounded and stabilizing solution of a discrete time Riccati equation with periodic coefficients were included in this paper published in Ann. Acad. Rom. Sci. Ser. Math. Appl. Vol. 7, No.1. Among the analyzed methods there is the Newton Kantorovich type method. Even if this iterative method is fast convergent, in the stochastic framework, it is hardly implementable, because, at each iteration we need to compute the unique periodic solution of a discrete-time generalized Lyapunov equation. In the afore mentioned work, we provided a method to compute efficiently the periodic solution of a discrete-time Lyapunov equation with periodic coefficients. It is worth mentioning that the main difficulty in computation of such a solution consists in the fact that we do not know apriori the initial conditions of the periodic solution. Involving the H-representation technique we have shown how a discrete time generalized Lyapunov equation with periodic coefficients can be transformed in nonhomogeneous linear equations with periodic coefficients on an euclidian space. For such an equation the initial conditions of the periodic solution can be obtained solving a system of standard linear equations.

7) **UNILATERAL CONDITIONS ON THE BOUNDARY FOR SOME SECOND ORDER DIFFERENTIAL EQUATIONS**, Dan Tiba, Ann. Acad. Rom. Sci. Ser. Math. Appl. Vol. 7, No. 1 (2015), p.121 - 136.

In elasticity, second order elliptic equations model the deformation of membranes (in dimension two). In the unilateral case contact problems are modeled and the equation is strongly nonlinear. This paper studies regularized variants, i.e nonlinear elliptic equations that do not involve jumps. Their ordinary differential and parabolic counterparts are also discussed. Existence results are proved under weak assumptions on the nonlinearity and are compared via examples with known results from the literature.

8) **An algorithm for the computation of the generalized solution for implicit systems**, Roxana Nicolai, Ann. Acad. Rom. Sci. Ser. Math. Appl. Vol. 7, No. 2 (2015), p.308 - 320.

The implicit parametrization method, studied for instance in the paper 1) from this report, offers the solution of degenerate implicit systems as a limit in the Hausdorff-Pompeiu metric. This approximation procedure depends very much on the choice of the initial conditions in the associated Hamiltonian systems. The present work proposes an algorithm and discusses the question of obtaining reliable results. There are many relevant numerical experiments and clear rules to be followed in such problems.

D) Conference articles

9) **SHAPE OPTIMIZATION AND THE IMPLICIT PARAMETRIZATION METHOD. APPLICATIONS**, Roxana Nicolai, Proceedings of the Humboldt Colloquium "On form and Pattern", Bucharest, June 2014, C.Vasilescu, M.Flonta, I.Craciun Eds., Editura Academiei, Bucharest (2015) p.83 - 97.

This paper discusses the implicit parametrization method and continues the investigations from the paper 1) above and from previous works of the authors. The applications in shape optimization are explained and new theoretical applications to implicitly defined equations (for instance of Lagrange type) or to variational methods associated to implicit functions, are introduced and analyzed. Numerical examples are also provided. A variational approach is discussed as well.