Scientific Report

On the implementation of the Grant 145/2012 in the period December 2013 - December 2014

A)Team Organization and main activities

In the setting of the Grant PN-II-ID-PCE-2011-3-0211, director CS 1 Dr. Dan Tiba (Institute of Mathematics, Romanian Academy, Bucharest) the activities of the year 2014 followed the plan from the project submitted during the Grants competition in 2011. The members of the team are CS 1 Dr. Dan Tiba (director), Prof. Dr. Andrei Halanay, (Univ. Politehnica, Bucharest) Ph. D. student Diana Merlusca (Institute of Mathematics, Romanian Academy, Bucharest). Starting with April 2012, CS 1 Dr. Vasile Dragan from the Institute of Mathematics, Romanian Academy, Bucharest, is as well a very active member of the Grant team. In the autumn of 2012, Roxana Nicolai (who finished her Master in July 2012) started her PhD studies at the Institute of Mathematics, Romanian Academy, Bucharest, supervisor CS 1 Dr. Dan Tiba and became a member of the Grant team from 2013. Unfortunately, we lost our colleague, Prof. V.Arnautu from the Univ. of Iasi, who deceased on January 4-th 2014, after a sudden illness.

The activity in the setting of the Grant has in 2014, as one of the main results, nine papers published or in an advanced publication stage (among which five are ISI, one is BDI and three are conference articles) by the members of the Grant team, alone or in collaboration with well known mathematicians as Luciano Pandolfi (Italy), Samir Aberkane and Cornel Murea (France), Ivan Ivanov (Bulgaria). All the articles are strictly related to the research themes of the Grant. Other articles are in an advanced stage of elaboration, and they will be reported in the next step of the Grant.

Diana Merlusca finished her Ph. D. studies in September 2014 with a very good thesis *Applications of the duality in some infinite dimensional optimization problems*, strongly related to the subjects and objectives of the Grant. She also got support from a POSDRU grant, for her work.

As prestigious international activities related to the Grant, we mention that Dan Tiba continued his work in the setting of the European Research Council, Bruxelles, chaired an Invited Session at the International Congress of Mathematicians, Seul, August 2014, delivered plenary invited talks at the International Workshop on *Variational and Hemivariational Inequalities: Theory, Numerics, and Applications*, June

28-30, 2014, at Xi'an Jiaotong University, at CAIM 2014, 17-21 September 2014, Univ. of Bacau. At the Humboldt Colloquium *On Form and Pattern* in Bucharest, May 2014, Dan Tiba and Roxana Nicolai had as well invited lectures. All these events were supported mainly from external funds.

With the support of the Grant, the members of our team attended and delivered lectures at the AIMS meeting in Madrid, August 2014 (Dan Tiba and Roxana Nicolai), at the 21st International Symposium on Mathematical Theory of Networks and Systems, July 2014, Groningen (Vasile Dragan), at the Hurwicz meeting, at the Banach Center in Warsaw, November 2014 (Dan Tiba). These dissemination activities are strongly related to the objectives of the Grant.

B) ISI papers

We discuss now the five ISI papers, the first three are published in 2014, for number 4 the galley proofs have been already done and the paper is planned to appear in DCDS, vol 35, no.6, 2015 and number 5 has already a positive report and submitted the revised version to JMAA.

1) **On computing the stabilizing solution of a class of discrete-time periodic Riccati equations**, by Vasile Dragan, Samir Aberkane and Ivan G. Ivanov, Int. J. Robust Nonlinear Control (2013), DOI: 10.1002/rnc.3131., impact factor 2,652.

In this paper, an iterative method for numerical computation of the bounded and stabilizing solution of a discrete-time Riccati equation (difference Riccati equation) with indefinite sign of its quadratic part, is proposed.

Shortly, the proposed method can be described as follows: for each $k = 0, 1, ...$ one constructs the periodic sequences $\{X^k(t)\}_{t\in\mathbf{Z}}$ and $\{Z^k(t)\}_{t\in\mathbf{Z}}$ by $X^0(t) = 0$ for all $t \in \mathbf{Z}$ and $X^k(t) = X^{k-1}(t) + Z^{k-1}(t), k \geq 1, t \in \mathbf{Z}$ while, for each $k = 0, 1, ...$, $Z^k(\cdot)$ is computed as the unique stabilizing and periodic solution of a suitable discrete-time Riccati equation with defined sign (for which already exist numerical methods).

We have shown that for each $t \in \mathbf{Z}$ the sequence $\{X^k(t)\}_{k\geq 0}$ is not decreasing and converges to the unique bounded and stabilizing solution of the discrete-time Riccati equation with indefinite sign under consideration and $\lim_{k\to\infty} Z^k(t) = 0$ for all $t \in \mathbf{Z}$ (here $\mathbf{Z}$ stands for the set of integers).

2)**Stabilizing solution of periodic game-theoretic Riccati differential equation of stochastic control**, Vasile Dragan, IMA Journal of Mathematical Control and Information (2014) Page 1 - 27, doi:10.1093/imamci/dnu026, impact factor 0,967.

In this paper, we have considered the problem of numerical computation of the stabilizing and periodic solution of a game theoretic Riccati differential equation with periodic coefficients of stochastic control. The proposed algorithm consists of numerical computation of two sequences of periodic functions $\{X^k(t)\}_{k\geq 0}$, $\{Z^k(t)\}_{k\geq 0}$, where, at each step $k = 0, 1, 2, ..., Z^k(\cdot)$ is the unique stabilizing and periodic solution of a Riccati differential equation with defined sign of its quadratic part (for which already exist specific methods for numerical computation). Further one takes $X^k(t) = X^{k-1}(t) + Z^{k-1}(t), k = 1, 2, ...$ with $X^0(t) = 0, t \in \mathbf{R}$. It is shown that $X^k(t)$ converges to the unique stabilizing and periodic solution of the game theoretic Riccati equation under consideration and $Z^k(t)$ tends to zero when k tends to ∞.

3) H_2 **optimal filtering for continuous-time periodic linear stochastic systems with state-dependent noise**, Vasile Dragan, Samir Aberkane, *Systems and Control Letters*, 66, (2014), pag. 35-42.

(http://dx.doi.org/10.1016/j.sysconle.2013.12.020 0167-6911/2014 Elsevier), impact factor 1,886.

In a problem of estimation of a signal generated by a dynamical system affected by multiplicative and additive white noise perturbations, the well known Kalman-Bucy filter cannot be implemented because its state space representation contains the same multiplicative white noise perturbations which affect the dynamical system whose output is estimated. In this work, we have studied the problem of the estimation of a signal which is an output of a dynamical system modeled by a system of Ito differential equations with multiplicative and additive white noise and periodic coefficients. The class of admissible filters consists of all deterministic systems with periodic coefficients of the form:

$$\dot{x}_f(t) = A_f(t)x_f(t) + B_f(t)y(t)$$

$$z_f(t) = x_f(t)$$

fed with the measurements $y(t)$ and having as output the estimated signal $z_f(t)$. The quality of estimation achieved by an admissible filter is measured by the mean square of the deviation of the estimated values $z_f(t)$ from the values $z(t)$ of the signal which must be estimated. we have shown that among all admissible filters of arbitrary dimension of their state space, the best estimation is achieved by a filter having the state space representation described based on the unique periodic solution of a suitable Lyapunov type differential equation and the unique stabilizing and periodic solution of a Riccati differential equation with positive sign of its quadratic part. The derivation of the state space representation of the optimal filter in this paper was done under the assumption that the dynamical system generating the signal which must be estimated is exponentially stable in mean square. It remains as a challenge for further research to obtain the optimal filter under some weaker assumptions than exponential stability in mean square.

4) **IMPLICIT FUNCTIONS AND PARAMETRIZATIONS IN DIMENSION THREE: GENERALIZED SOLUTIONS**, Dan Tiba, Roxana Nicolai, DISCRETE AND CONTINUOUS DYNAMICAL SYSTEMS Volume 35, Number 6, June 2015, doi:10.3934/dcds.2015.35.xx, impact factor 0,923.

Fixed domain methods in shape optimization involve the use of implicit representations of unknown domains. In problems with Dirichlet boundary conditions, there are already techniques to handle such approaches. In the case of Neumann (or other types of boundary conditions) a very detailed knowledge of the unknown boundaries is necessary. This paper discusses a special new approach, based on ordinary differential equations, in order to obtain the desired data. Starting from the implicit equation (in dimension three) an iterated Hamiltonian system is introduced that provides the local solution of the equation in a parametric form. This allows to handle for instance the normal derivatives on the unknown boundaries. The approach can be extended to the critical case, that could not be handled up to now in the mathematical literature. A notion of generalized solution is defined for singular equations. Several numerical examples, both in the classical and in the critical cases, are indicated. The article extends previous work of the authors and will be generalized to general implicit systems in arbitrary dimension, in future papers.

5) **Approximate controllability and lack of controllability to zero of the heat equation with memory**, Andrei Halanay, Luciano Pandolfi (accepted modulo revision) Journal of Mathematical Analysis and Applications (2014), impact factor 1,119.

In this paper we consider the heat equation with memory in a bounded region $\Omega \subset \mathbb{R}^d$, $d \geq 1$, in the case that the propagation speed of the signal is infinite (i.e. the Colemann-Gurtin model). The memory kernel is of class C^1. We examine its controllability properties both under the action of boundary controls or when the controls are distributed in a subregion of Ω. We prove approximate controllability of the system and, in contrast with this, we prove the existence of initial conditions which cannot be steered to hit the target 0 in a certain time T, of course when the memory kernel is not identically zero. In both the cases we derive our results from well known properties of the (memoryless) heat equation.

The results of this paper enter the objectives of the present Grant.

C) BDI Articles

6) **Lack of controllability of thermal systems with memory**
Andrei Halanay and Luciano Pandolfi, EVOLUTION EQUATIONS AND CON-

TROL THEORY Volume 3, Number 3, September 2014 pp. 485–497, doi:10.3934/eect.2014.3.485

Heat equations with memory of Gurtin-Pipkin type

$$w_t = \alpha w_{xx} + M(t-s) w_{xx}(x,s)s\,, \qquad w(x,0) = \xi(x), \tag{1}$$

and where the time $t = 0$ is the time after which a boundary control f is applied to the system, i.e. where we assume the boundary conditions

$$w(0,t) = f(t)\,, \quad w(\pi,t) = 0 \quad t > 0\,.$$

are studied. It appears that when $\alpha = 0$ they have controllability properties which strongly resemble those of the wave equation. Instead, recent counterexamples show that when $\alpha > 0$ the control properties do not parallel those of the (memoryless) heat equation, in the sense that there are square integrable initial conditions which cannot be controlled to zero. The proof of this fact, in previous papers, consists in the construction of two quite special examples of systems with memory which cannot be controlled to zero. Here we prove that, in one spatial dimension, lack of controllability holds in general, for every smooth memory kernel $M(t)$.

The research of the second author fits the plans of INDAM-CNR and of the project “Groupement de Recherche en Contrôle des EDP entre la France et l’Italie (CONEDP)”

D) Conference articles

7) **Steady Fluid-Structure Interaction Using Fictitious Domain**, Andrei Halanay, Cornel Murea, in C. Potzsche et al. (Eds.): CSMO 2013, IFIP AICT 443, pp. 110, 2014. DOI: 10.1007/978-3-662-45504-3 12

Let $D \subset \mathbb{R}^2$ be a bounded domain with smooth boundary ∂D. Let Ω_0^S be the undeformed structure domain, and suppose that its boundary admits the decomposition $\partial\Omega_0^S = \Gamma_D \cup \Gamma_0$, where Γ_0 is a relatively open subset of the boundary. On Γ_D we impose zero displacement for the structure. We assume that $\overline{\Omega}_0^S \subset D$.

Suppose that the structure is elastic and denote by $\mathbf{u} = (u_1, u_2) : \Omega_0^S \to \mathbb{R}^2$ its displacement. A particle of the structure with initial position at the point $\mathbf{X}$ will occupy the position $\mathbf{x} = \varphi(\mathbf{X}) = \mathbf{X} + \mathbf{u}(\mathbf{X})$ in the deformed domain $\Omega_u^S = \varphi\left(\Omega_0^S\right)$.

We assume that $\overline{\Omega}_u^S \subset D$ and the fluid occupies $\Omega_u^F = D \setminus \overline{\Omega}_u^S$.

We set $\Gamma_u = \varphi(\Gamma_0)$, then the boundary of the deformed structure is $\partial\Omega_u^S = \Gamma_D \cup \Gamma_u$ and the boundary of the fluid domain admits the decomposition $\partial\Omega_u^F = \partial D \cup \Gamma_D \cup \Gamma_u$.

This problem is approached using fictitious domain technique with penalization. Numerical results are presented.

8) **A Penalization Method for the Elliptic Bilateral Obstacle Problem**, Dan Tiba, Cornel Murea, in C. Potzsche et al. (Eds.): CSMO 2013, IFIP AICT 443, pp. 110, 2014. DOI: 10.1007/978-3-662-45504-3 18

In this paper we propose a new algorithm for the wellknown elliptic bilateral obstacle problem. Our approach enters the category of fixed domain methods and solves just linear elliptic equations at each iteration. The approximating coincidence set is explicitly computed. The algorithm represents an application of methods used in shape optimization to free boundary problems. This has a natural meaning since both type of problems involve unknown geometries. In the numerical examples, the algorithm has a fast convergence.

9) **SHAPE OPTIMIZATION AND THE IMPLICIT PARAMETRIZATION METHOD. APPLICATIONS**, Roxana Nicolai (accepted for publication in Proceedings of the Humboldt Colloquium "On form and Pattern", Bucharest, June 2014), Editura Academiei, Bucharest (2015).

This paper discusses the implicit parametrization method and continues the investigations from the paper 4) above and from previous works of the authors. The applications in shape optimization are explained and new theoretical applications to implicitly defined equations (for instance of Lagrange type) or to variational methods associated to implicit functions, are introduced and analyzed. Numerical examples are also provided.